

Math 135 Formula Sheet

Linear Equations

$$\text{Slope: } m = \frac{y_2 - y_1}{x_2 - x_1}$$

slope intercept form: $y = mx + b$

point slope form: $y - y_1 = m(x - x_1)$

Augmented Matrix

$$\begin{array}{cc|c} \text{Coefficient} & \text{Constraint} & \\ \hline \left[\begin{array}{cc|c} a_{11} & a_{12} & k_1 \\ a_{21} & a_{22} & k_2 \end{array} \right] \end{array}$$

Gauss-Jordan Elimination

One Solution

Infinite Solutions

No Solution

$$\left[\begin{array}{cc|c} 1 & 0 & m \\ 0 & 1 & n \end{array} \right]$$

$$\left[\begin{array}{cc|c} 1 & m & n \\ 0 & 0 & 0 \end{array} \right]$$

$$\left[\begin{array}{cc|c} 1 & m & n \\ 0 & 0 & p \end{array} \right]$$

Matrix Addition

Matrices must be same size

$$[m \times n] + [m \times n] = [m \times n]$$

$$[m \times n] + [m \times p] = \text{undefined}$$

Matrix Multiplication

End of one matrix must match the beginning of the next matrix

$$[m \times n] \cdot [n \times p] = [m \times p]$$

Match

$$[m \times n] \cdot [p \times q] = \text{Undefined}$$

No Match

Inverse Matrix

$$[M | I] \Rightarrow [I | M^{-1}]$$
$$\left[\begin{array}{cc|cc} a & b & 1 & 0 \\ c & d & 0 & 1 \end{array} \right] \Rightarrow \left[\begin{array}{cc|cc} 1 & 0 & e & f \\ 0 & 1 & g & h \end{array} \right]$$

Geometric Method for Solving Linear Programming Problem

- Find feasible region by graphing your constraints.
- Determine if a minimum or maximum exists.
- Find coordinates for the corners of the feasible region.
- Construct a corner point table.

- Determine the optimal solution.
- Interpret the solution(s).

Simplex Tableau

The number of constraint equals the number of slack variables.

$$\begin{array}{c}
 s_1 \\
 s_2 \\
 P
 \end{array}
 \begin{array}{c}
 x_1 \quad x_2 \quad s_1 \quad s_2 \quad P \\
 \left[\begin{array}{ccccc|c}
 a_{11} & a_{12} & 1 & 0 & 0 & k_1 \\
 a_{21} & a_{22} & 0 & 1 & 0 & k_2 \\
 -P_1 & -P_2 & 0 & 0 & 1 & 0
 \end{array} \right]
 \end{array}
 \qquad
 \begin{array}{c}
 s_1 \\
 s_2 \\
 P
 \end{array}
 \begin{array}{c}
 x_1 \quad x_2 \quad x_3 \quad s_1 \quad s_2 \quad P \\
 \left[\begin{array}{cccccc|c}
 a_{11} & a_{12} & a_{13} & 1 & 0 & 0 & k_1 \\
 a_{21} & a_{22} & a_{23} & 0 & 1 & 0 & k_2 \\
 -P_1 & -P_2 & -P_3 & 0 & 0 & 1 & 0
 \end{array} \right]
 \end{array}$$

Sets

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

$$= \{x \in U \mid x \notin A\}$$

Counting Methods

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Multiplication: If n operations (O_1, O_2, \dots, O_n) are performed, then there are $N_1 \cdot N_2 \cdot \dots \cdot N_n$ outcomes.

Combinations and Permutations

$$0 \leq r \leq n$$

$${}^n C_r = \frac{n!}{r!(n-r)!} \qquad {}^n P_r = \frac{n!}{(n-r)!}$$

Arrangement Irrelevant

Order Specific

Probabilities, Odds and Expected Values

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$

$$P(A \cap B) = P(A) \cdot P(B \mid A) = P(B) \cdot P(A \mid B)$$

$$\text{Independent Events: } P(A \cap B) = P(A) \cdot P(B)$$

$$E(x) = x_1 P_1 + x_2 P_2 + \dots + x_n P_n$$

$$\text{Odds for an Event (E)} = \frac{P(E)}{P(E')}$$

$$\text{Odds against an Event (E)} = \frac{P(E')}{P(E)}$$

Markov Chains

$$P = \begin{matrix} & A & A' \\ A & [AA & AA'] \\ A' & [A'A & A'A'] \end{matrix}$$

$$\text{Initial State: } S_0 = [A \quad A']$$

$$\text{First State: } S_1 = S_0P$$

$$\text{Second State: } S_2 = S_1P$$

⋮

$$k - \text{th State: } S_k = S_{k-1}P$$

$$\text{Powers of Transition Matrix: } S_k = S_0P^k$$

A transition matrix P is regular if some power of P has only positive entries.

A Markov chain is a **regular Markov chain** if its transition matrix is regular.

Means and Measures of Dispersion

$$\text{Mean (Ungrouped): } \text{Mean} = \frac{\sum_{i=1}^n x_i}{n}$$

$$\text{Standard Deviation (Ungrouped): } SD = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$$

$$\text{Mean (Grouped): } \text{Mean} = \frac{\sum_{i=1}^n x_i f_i}{n}$$

$$\text{Standard Deviation (Grouped): } SD = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2 f_i}{n-1}}$$

$$\text{Mean} = \bar{x} \text{ (sample) or } \mu \text{ (population)}$$

$$SD = s \text{ (sample) or } \sigma \text{ (population)}$$

Binomial Distribution

$$\mu = np \quad \sigma = \sqrt{npq}$$

z-Score

$$z = \frac{x - \mu}{\sigma}$$
